Homework 3

Deadline: June 8, 2023

1. (Sec.4.2.1) Sketch
$$k_N(r) = \frac{\Gamma[\frac{1}{2}(N-1)]}{\Gamma(\frac{1}{2}N-1)\sqrt{\pi}}(1-r^2)^{\frac{1}{2}(N-4)}$$
for $(a)N=3,(b)N=4,(c)N=5$ and $(d)N=10$.

2. (Sec.4.2.1) Suppose a sample correlation of 0.65 is observed in a sample of 10. Test the hypothesis of independence against the alternatives of positive correlation at significance level 0.05.

3. (Sec. 4.2.3) Use Fisher's z to test the hypothesis $\rho = 0.7$ against alternatives $\rho \neq 0.7$ at the 0.05 level with $r = 0.5$ and $N = 50$.

4. (Sec. 4.2) Prove that if Σ is diagonal, then the sets r_{ij} and a_{ii} are independently distributed. [Hint: Use the facts that r_{ij} is invariant under scale transformations and that the density of the observations depends only on the a_{ii}.]

5. (Sec. 5.2.2) Let $T^2 = N\bar{\mathbf{x}}^\top \mathbf{S}^{-1}\bar{\mathbf{x}}$, where $\bar{\mathbf{x}}$ and $\mathbf{S}$ are the mean vector and covariance matrix of a sample of N from $N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. Show that T^2 is distributed the same when $\boldsymbol{\mu}$ is replaced by $\boldsymbol{\lambda} = (\tau, 0, ..., 0)^\top$, where $\tau^2 = \boldsymbol{\mu}^\top \boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ is replaced by $\mathbf{I}$.

6. (Sec. 5.3) Use the data in Section 3.2 to test the hypothesis that neither drug has a soporific effect at significance level 0.01.

7. (Sec. 5.3) Let $\mathbf{x}_\alpha^{(i)}$ be observations from $N(\boldsymbol{\mu}^{(i)}, \boldsymbol{\Sigma}_\mathbf{i}), \alpha = 1, \ldots, N_i, i = 1, 2$. Find the likelihood ratio criterion for testing the hypothesis $\boldsymbol{\mu}^{(1)} = \boldsymbol{\mu}^{(2)}$.